## THE 47th—19th INTERNATIONAL—RUDOLF ORTVAY

## PROBLEM SOLVING CONTEST IN PHYSICS

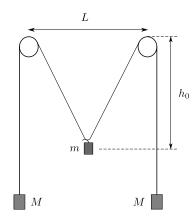
## 28 October— 7 November 2016

1. Generally the front wheels of a tractor are much smaller than the rear ones. Nowadays the factories frequently construct tractors with four big wheels. What was the advantage of the small front wheels, and what is the cause of the change that the front wheels increased? What is the advantage of the large wheels? (The tractors have always rear wheel drive.)



(Géza Tichy)

- 2. Let us construct a slingshot which projects vertically. The plan is shown on the Figure: a rope is lead over two fixed pulleys, the latter ones are at distance of L, with two masses M at same heights at the ends. Assume that the diameter of the pulleys is very small relative to L. To the middle of the rope, a third mass m is hooked, at a depth of  $h_0$  below the level of the pulleys. Let's launch the slingshot by releasing the mass in the middle.
  - a) In the first experiment, by accident, the parameters Q = M/m and  $x_0 = h_0/L$  were wrongly chosen, so that instead of starting upwards, the mass m moved down. Which is this region of the parameters?
  - b) In the second case, at a given value Q the middle mass did not move at all. Well let's try with a small push: the projectile starts to oscillate. Determine the frequency of the small oscillations.
  - c) Progressing with the experimental setup, Q and  $x_0$  are chosen such that the slingshot acts as a true launcher device. Determine the velocity of the projectile mass at the instance when it detached from the rope.
  - d) Let us give a try with the following parameters: starting level  $h_0 = 2L$ , and equal masses M = m. How long do the release takes, that is, how much time is needed after release for the projectile to be detached? In the same experiment, determine the value of depth h where the projectile's velocity is maximal. What is the maximal value of the velocity?



(Péter Simon and András Csordás)

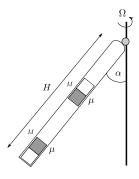
3. The two rails of a symmetric step ladder are of mass m, and held together by a hinge on the top. We fix a mass M on the top of the ladder. At the instance t=0, the spreader rope fixing the two rails breaks, and the ladder legs slip apart. Determine how much time it takes to reach the ground as a function of the mass ratio Q=M/m and the initial opening angle between the rails  $2\alpha_0$ . How much is the velocity of the object with mass M at the point of impact? Assume that the rails of the ladder are homogeneous rods, and the lower ends slip on the ground without friction.

(József Cserti)

- 4. A thin tube is attached next to a high tower. We let a heavier ball fall down into the tube, and some time later we release a smaller ball after the first. The small ball collides with the bigger after the latter bounced back up from the ground, thus hurling the small ball upwards. Assume that the collisions are perfectly elastic and that air drag and friction are negligible. (The mere purpose of the tube is to ensure that falling balls are guided along a vertical line, without exerting any friction.)
  - a) Give an upper limit how high, relative to the height of the tower, the smaller ball can reach this way!
  - b) Assume that after the second ball, we release a third, even smaller ball from the tower, which bounces off the second ball on its way up. This may be continued with more balls. How high the n-th ball can reach? Which are the ideal time intervals for the releases to approximate the maximum height?
  - c) Assume that the mass of each subsequent ball is half of the mass of the one before. If the balls are released from the height of the Eiffel Tower, how many of them are needed to reach the Earth's escape velocity?
  - d) How much is the efficiency of this 'rocket launcher' device? That is, which fraction of the potential energy of the initial state will be transferred to the last ball leaving the Earth?

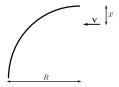
(Gyula Dávid)

- 5. A small magnet of mass M with magnetic moment  $\mu$  is fixed to the lower end of a thin glass tube. The length of the tube is H and its mass is negligible. The magnetic moment vector points in the direction of the tube. Within the tube, a same kind of other small magnet piece, turned in opposite direction relative to the first one, can slide without friction. The upper end of the tube is attached to a vertical shaft. The shaft is rotated around the vertical axis with a constant angular velocity  $\Omega$ . The value of the gravitational acceleration is g.
  - a) The upper end of the tube is fixed at a constant angle  $\alpha$  relative to the direction of the shaft. Where is the equilibrium position of the moving magnet, and which is the frequency of the oscillations around this equilibrium position?
  - b) In this case, the upper end of the glass tube is fixed with a frictionless hinge to the shaft. Where is (are) the equilibrium position(s) of the system, and which kind of oscillations can take place around the equilibrium state(s)?



(József Cserti and Gyula Dávid)

6. A mass point approaches an arc-shaped wall (quarter of a circle) standing on a horizontal plane. The mass point moves on the horizontal plane with initial velocity v and with direction parallel to the initial portion of the wall at distance x from the edge of the wall. Assume that friction is negligible. How many times the object will collide with the wall, and what is the final velocity of the mass point if the collisions are a) perfectly elastic, b) perfectly inelastic? Investigate in details the limit case  $x \to 0$ .



(Géza Tichy)

7. A special dice has the property that the probability of throwing 1 is 50 %, whereas all the other numbers appear with the probability of 10 % each. Can we make any statements about the properties of the dice, such as center of mass, or tensor of inertia?

(Zsolt Bagoly)

8. Let us analyze the jump by Felix Baumgartner based on the informations available on the websites below:

https://youtu.be/raiFrxbHxV0,

http://oroszl.web.elte.hu/szamprob/notebooks/Package05/data/BAUMGARTNER/

Let us prepare the simplest model which explains the data to the highest precision! Consider

- the variation of the properties of the atmosphere as a function of height,

- the changes in the orientation and the shape of the falling body.

Discuss in detail the main sections of the fall. How the outcome could have changed, if, instead of a living person, a pupper falls with uncontrollable limbs?

(László Oroszlány)

9. There are a number of physics textbooks which explain the Doppler effect with the example of the ambulance car: check for the tone of the sound from an ambulance which approaches directly towards you, and then observe the deeper tone as it moves away. Unfortunately, those who tried to experiment this way, fared ill, since the ambulance car ran over them.

Let us try to be more careful, then! Let us step away from the straight trajectory of the ambulance, which runs at a constant speed V < c (c is the speed of sound), to a distance of H. Calculate the time dependence of the observable frequency of the sound generated by the siren running at constant frequency  $\omega_0$ . Let us plot this function. Study the case of the supersonic ambulance, V > c, as well.

In fact mindful physicist consider not only the tone, but the direction of the sound as well. At the same time, visually the true direction of the ambulance can be followed (we can assume that the speed of light is infinitely large). Plot the the heard direction angle of the ambulance as a function of the seen direction angle. Discuss the cases of subsonic (V < c), supersonic (V < c) and the sonic (V = c) speeds.

(Gyula Dávid)

10. In a perfectly heat insulated kitchen, we switch on a refrigerator. How do the average electrical power consumption of the fridge change as a function of time? Let us assume that the refrigerator works as an ideally efficient heat pump!

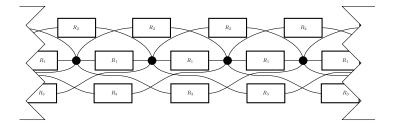
(András Bodor)

11. Consider a vertical cylindrical tube of thin wall which stands on very thin legs of negligible air drag. The cylinder is heated by an internal (electric or hot water) system, such that the temperature of the inner wall of the tube is all the time higher than the surrounding air. The outer part of the tube is heat insulated, that is, the heating makes no effect there.

The air inside the cylinder starts to move upwards. Where do the vertical momentum for the rising air come from?

(Zoltán Hlavathy)

12. Let us consider a one dimensional periodic resistor network where we insert  $R_i$  resistances between nodes depicted by black dots. The resistor between first neighbour is  $R_1$ , between second nearest neighbors it is  $R_2$  and so on. What is the resultant resistivity between two arbitrary nodes if we only consider first and second nearest neighbour? What is the resultant resistivity if the first N connections are considered? If a numerical solution is presented discuss the details of the applied algorithm.

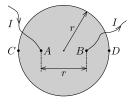


(László Oroszlány)

13. Somewhere in a galaxy far, far away, exists an object made of resistors with resistance R. The resistors connect the nearest neighbour points of the lattice with position vectors  $\mathbf{r} = n_x \mathbf{e}_x + n_y \mathbf{e}_y + n_z \mathbf{e}_z$ . Here,  $n_x, n_y, n_z \in \mathbb{Z}$  and  $0 \le n_z \le D$ , furthermore,  $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$  are unit vectors in the direction of the x, y, and z axes. Ants living on the surface  $(n_z = 0)$  of the object like to determine the width D of their world, hence they measure the effective resistance  $R_b$  between two neighbouring points. Help to the ants to determine the function  $R_b(R, D)$ . Discuss the special cases D = 0 and  $D \to \infty$ .

(Gábor Széchenyi and József Cserti)

14. Taking a metallic disc of radius r and thickness d ( $d \ll r$ ) according to the figure, we feed electric current I at point A, and draw the same current from point B. Which is the value of the voltage between points C and D? The specific resistivity of the material of the disc is  $\varrho$ .



(Máté Vigh)

15. In anisotropic but homogeneous materials the dielectric coefficient  $\varepsilon$  is a tensor quantity. What is the equation of equipotent surfaces of a uniformly electrically charged infinite straight line?

(Géza Tichy)

16. A droplet of mercury with radius R floats in zero gravity. If the droplet is placed into a weak homogeneous electric field  $E_0$ , it will slightly elongate in the direction of the field lines. Determine the equilibrium shape of the droplet! The surface tension of mercury is  $\alpha$ , and let us assume that  $\varepsilon_0 E_0^2 R \ll \alpha$ !

(Máté Vigh)

- 17. Let's study charge configurations whose electric field is nowhere zero. A single charge is the simplest example. Two point charges also form such a configuration if their total charge (the zeroth multipole moment) vanishes.
  - a) Consider three collinear equally spaced point charges -q, 2q, -q. Notice that this configuration has vanishing zeroth and first multipole moments, that is, zero total charge and dipole moment. Show that the electric field of this configuration is nowhere zero.
  - b) Show if the electric field of n collinear or even coplanar point charges is nowhere zero, then their first (n-2) multipole moments all vanish.
  - c) For each n, find a configuration of n charges whose first (n-2) multipole moments are zero. Are there any such configurations which are not collinear?

(Zoltán Zimborás and Szilárd Farkas)

18. Carbon nanotubes are examples of quasi one-dimensional systems existing in nature. In semi-conducting nanotubes the motion of the electrons is restricted along the nanotube and electrons possess an effective mass  $m^*$ . The Coulomb interaction of the electrons is somewhat screened by the substrate on which the nanotube is laid, an effect usually taken into account by an increased relative permittivity,  $\epsilon \approx 2-4$ . External gates typically create a close to parabolic confinement potential for the electrons confined on the nanotube.

Let us therefore model the nanotube as an infinite one-dimensional system of electrons of effective mass  $m^*$ , confined in a parabolic potential, and interacting through a Coulomb interaction in a medium of permittivity  $\epsilon$ .

Treat the electrons classically, and determine the energy of the ground state configuration of N electrons on the tube as a function of N. Determine also the spatial distribution of charges. Analyze the large N limit. You can also derive analytical results for N=2 and N=3.

(Gergely Zaránd)

19. I'd like use on my monitor the most accurate color picture of the dawn (the sky at sunrise in last second before appearing of the Sun on the horizon). Let's take the ideal case, where the previous rains cleared the dust from the air, also there are no clouds. The lower side of monitor is the horizon ±90° around the Sun position, the center of upper side is the vertical 90° (zenith). I'd like to get the RGB values of pixels.



(Imre Sánta)

20. We illuminate a corner cube (corner reflector) of diameter 60 mm, located at distance of 3 km from us, by a laser of wavelength 532 nm, diameter 1 mm,  $TEM_{00}$  beam mode parameters. What is the intensity distribution of reflected beam around the laser?



(Imre Sánta)

21. The creation and propagation of sound is described by the inhomogeneous D'Alembert equation:

$$\frac{1}{c^2} \frac{\partial^2 u(t, \mathbf{r})}{\partial t^2} - \Delta u = f(t, \mathbf{r})$$

Assume that the observer is in the origin of the three dimensional space. A pointlike sound source moves on a straight line at a minimal distance a from the origin at velocity V.

Calculate the sound intensity at the origin, that is, the solution of the D'Alembert equation as a function of time. Study the cases of subsonic (V < c), supersonic (V > c) and sonic (V = c) speeds, as well as the limiting cases  $V \to 0$ ,  $V \to c \pm 0$  and  $V \to \infty$ .

If we do have some energy left, let us repeat the calculation for the cases of 1 + 2 and 1 + 4 dimensional space-times as well.

(Gyula Dávid)

22. Next to a photovoltaic rod of length L, there is a rod-shaped light source of the same length. Assume that the rods are parallel. From the two ends of the photovoltaic rod, wires are connected to a current meter which records the photogenerated current. Just in front of the photovoltaic rod, a thin plate of length 2L passes: that is, at rest the plate would fully cover the light sensor. The velocity of the plate is  $12/13\,c$ .

How do the current reading as a function of time look like? Will there be an interval of time when the current goes to zero?

Describe the phenomenon from the reference system fixed to the moving plate as well.

(Gyula Dávid based on the idea of Péter Skirka)

23. A relativistic particle orbits in a static, spherically symmetric four-scalar field, created by a source fixed in the origin. The rest mass of the particle is generated fully by the scalar field. Determine the value of the scalar field as a function of the radius, provided that the trajectory of the particle is an ellipse, with the origin in the a) middle, b) focal point of the ellipse!

(Gyula Dávid)

24. Two celestial objects of mass  $m_1$  and  $m_2$  are orbiting around each other on a very elongated ellipse ( $\varepsilon \approx 1$ ). The system is isolated, there is no other proximate celestial body. Let us study how do the parameters of the elliptical orbit change as a consequence of radiation of gravitational waves. Estimate the characteristic time of the process which transfers the elongated ellipse to a nearly circular orbit.

(Gyula Dávid)

25. Werner Heisenberg and John von Neumann are studying a particle confined in the one dimensional interval [-a, a] with a mass of m = 1/2. The Hamiltonian of the particle is  $\hat{H} = -\nabla^2$   $(\hbar \equiv 1)$ , and its wavefunction takes the form of  $\psi(x) = \sin(x^2 - a^2)/\sqrt{N}$ , where N is a normalization constant. The two scientists are interested in the validity of the energy-time uncertainty relation in the state above, and in order to get the standard deviation of the energy, they calculate the following matrix elements:

$$i) \int_{-a}^{a} \psi^{*}(x)(-\nabla^{2})\psi(x) dx$$

$$ii) \int_{-a}^{a} \psi^{*}(x)\nabla^{4}\psi(x) dx.$$

At a certain value of a Heisenberg gets really upset. What is this value and how does John von Neumann calm him down?

(Gergely Fejős)

26. Carbon nanotubes are examples of quasi one-dimensional systems existing in nature. In semi-conducting nanotubes the motion of the electrons is restricted along the nanotube and electrons possess an effective mass  $m^*$ . The Coulomb interaction of the electrons is somewhat screened by the substrate on which the nanotube is laid, an effect usually taken into account by an increased relative permittivity,  $\epsilon \approx 2-4$ . External gates typically create a close to parabolic confinement potential for the electrons confined on the nanotube.

Let us therefore model a carbon nanotube as an infinite one-dimensional system of electrons of effective mass  $m^*$ , confined in a parabolic potential, and interacting through a Coulomb interaction in a medium of permittivity  $\epsilon$ .

Determine quantum mechanically the ground state configuration (wave function and corresponding charge density), and the energy of two confined electrons. Determine also the first excited state and its spin and energy. The splitting of these states can be identified as the exchange coupling, J. Determine J as a function of the strength of the parabolic confinement. For the sake of simplicity, assume that electrons have a spin S=1/2. (In reality, electrons on nanotubes possess also a chirality quantum number,  $\tau=\pm 1$ , specifying their angular momentum around the nanotube.)

(Gergely Zaránd)

27. The Brouwer-Büttiker-Pretre-Thomas formula gives the charge  $dq_l$  pumped into lead l by an adiabatically slowly changing scatterer in the infinitesimal time interval  $d\tau$  as

$$dq_l = \frac{i}{2\pi} \operatorname{Tr} \left[ P_l \, \partial_{\tau} S(\tau) \, S^{\dagger}(\tau) \right] d\tau \,,$$

where S is the scattering matrix and the matrices  $P_l$  project the scattering matrix onto the portion corresponding to lead l, that is:

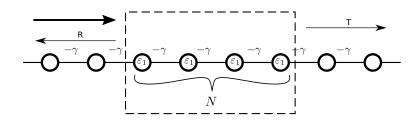
$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}, \quad P_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

a) Construct a model of the simplest one dimensional scattering process where the scattering matrix is independent of the energy of the incoming particles and where the scattering process can be described by at least two parameters.

b) According to Avron and coworkers if certain criteria are met adiabatic quantum pumps transfer integer charge in a pumping cycle (https://arxiv.org/pdf/math-ph/0105011v2.pdf). Under which adiabatic cycle will the pumped charge be an integer in the model devised in part a)?

(László Oroszlány)

- 28. Consider a one dimensional scattering problem on a simple tight-binding chain. The hopping between all lattice sites is  $-\gamma$ , the on-site potential is zero everywhere except in the scattering region where it is  $\varepsilon_1$ . The scatterer consists of N lattice sites.
  - a) Calculate the transmission probability of particles with E=0 impending on the scattering region as the function of N and  $\varepsilon_1$  in the parameter interval  $0 < \varepsilon_1/\gamma < 0.1$ .
  - b) Derive a low energy effective model with the help of the envelope-function approximation. Explain the systematic behavior of the transmission as the function of N with the help of the derived effective model.



(László Oroszlány)

29. We have 1000 independent radioactive nuclei in a box at zero time, t=0. The half life of each is 10 hours. Their daughter element is not radioactive. At what time can we state at 66 % confidence level that the last nucleus is already decayed?

(Ákos Horváth)

30. The quantum theory of the massive scalar field in 1+1 dimensions is described by the Hamiltonian  $(\hbar=c=1)$ 

$$H = \int_{-\infty}^{\infty} : \frac{1}{2} \left( \frac{\partial \varphi}{\partial t} \left( x, 0 \right) \right)^{2} + \frac{1}{2} \left( \frac{\partial \varphi \left( x, 0 \right)}{\partial x} \right)^{2} + \frac{m^{2}}{2} \varphi^{2} \left( x, 0 \right) :_{m} + : V \left( \varphi \left( x, 0 \right) \right) :_{m} dx$$

where

$$\varphi(x,t) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi\omega_k}} \left( a(k) e^{i(kx-\omega_k t)} + a^{\dagger}(k) e^{-i(kx-\omega_k t)} \right) dk$$

and where the operators  $a^{\dagger}(k)$  and a(k) create and annihilate particles of momentum k and mass m and satisfy the commutation relation

$$\left[a\left(k\right), a^{\dagger}\left(k'\right)\right] = \delta\left(k - k'\right).$$

Moreover,  $\omega_k = \sqrt{m^2 + k^2}$ , and the colons : :<sub>m</sub> indicate normal ordering with respect to m, with the understanding that creation operators are always on the left of annihilation operators in the enclosed expression.

Determine the Hamiltonian of the *same* system if the normal ordering is performed with respect to a different free boson of mass  $\mu \neq m$ , that is,

$$H = \int_{-\infty}^{\infty} : \frac{1}{2} \left( \frac{\partial \varphi}{\partial t} \right)^2 + \frac{1}{2} \left( \frac{\partial \varphi}{\partial x} \right)^2 + \frac{\mu^2}{2} \varphi^2 :_{\mu} + : \tilde{V} (\varphi) :_{\mu}$$

Give the function  $\tilde{V}(\varphi)$  exactly, corresponding to a)  $V(\varphi) = g\varphi^{2n}$ , b)  $V(\varphi) = \frac{1}{b^2}\cos(b\varphi)$ . (One may drop the constant term appearing inside the integral during the change of the scheme.)

(Márton Kálmán Lájer)

31. Let us consider two one dimensional coupled harmonic oscillators with unit mass described by the following Hamiltonian:

$$H = \frac{p_1^2}{2} + \frac{p_2^2}{2} + \frac{\omega_1^2}{2}(x_1^2 + x_2^2) + \frac{\omega_2^2}{2}(x_1 - x_2)^2.$$

The system is in its ground state, but one of the oscillators is completely invisible, and we can only perform measurements on the other one. What is the temperature of the latter?

(Gergely Fejős)

32. Consider the field theory of two interacting scalar fields  $\phi$  and  $\chi$  with the following Lagrangian:

$$\mathcal{L} = \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} m^2 \phi^2 + \frac{1}{2} (\partial \chi)^2 - \frac{1}{2} M^2 \chi^2 - g \phi^2 \chi.$$

The particle  $\chi$  can decay to two particles  $\phi$ . Show that the expected lifetime of the particle  $\chi$  takes the form

$$\langle T \rangle = \frac{-M}{\operatorname{Im}\left(\Pi(M^2)\right)},$$

where  $-i\Pi(p^2)$  denotes the eigenenergy of the particle  $\chi$ .

(Gábor Homa and László Lisztes)

33. Negative absolute temperature states may exist in systems where the energy is bounded from above. For such states the entropy decreases with increasing energy. Consider such an N-particle system with Hamiltonian

$$H = -\frac{J}{2} \sum_{i,j=1}^{N} \left[ (\boldsymbol{n}_i \cdot \boldsymbol{n}_j)^2 - \frac{1}{3} \right]$$

where  $n_i$  are 3-dimensional unit vectors for each i and J > 0. The equations of motion are

$$\frac{d\boldsymbol{n}_i}{dt} = \frac{\partial H}{\partial \boldsymbol{n}_i} \times \boldsymbol{n}_i = -J \sum_{i=1}^{N} (\boldsymbol{n}_i \cdot \boldsymbol{n}_j) (\boldsymbol{n}_j \times \boldsymbol{n}_i).$$

It is simple to show, that  $|\mathbf{n}_i| = 1$  is conserved for each i, and in addition to the total energy,  $\mathbf{L} = \sum_i \mathbf{n}_i$  is conserved. In the limit  $N \to \infty$ , the system is characterized by its number-density defined on the surface of the unit-sphere,  $\rho(\theta, \phi)$ . Determine the axially symmetric (with respect to  $\mathbf{L}$ ) statistical equilibrium  $\rho(\theta)$  which maximizes the Boltzmann entropy

$$S = -k \int_{S_2} \rho \ln \rho \ d\Omega$$

for fixed total energy,  $\boldsymbol{L}$ , and N. Plot the order parameter

$$Q = \int_{S_2} \left(\cos^2 \theta - \frac{1}{3}\right) \rho \, d\Omega$$

as a function of temperature for  $|\mathbf{L}| = 0$  and 0.4 N, respectively. Are the negative temperature equilibria stable?

(Bence Kocsis)

34. It is often said that in covariantly gauge fixed quantum electrodynamics, gauge invariance is encoded in the Ward identities. For instance, let us consider the well-known identity between  $\Gamma_{A^{\mu}\phi^{\dagger}\phi} \equiv \Gamma^{\mu}$  and  $\Gamma_{\phi^{\dagger}\phi} \equiv \Gamma$  proper vertices:

$$q_{\mu}\Gamma^{\mu}(q,p) = \Gamma(p) - \Gamma(p+q),$$

where  $A^{\mu}$  is the four-potential,  $\phi$  is a complex scalar field, and the gauge coupling has been set to unity,  $e \equiv 1$ . This identity (formally) can be derived from the 1PI projection of the Fourier transform of the divergence of the correlator  $\langle 0|T(j^{\mu}(x)\phi(y)\phi^{\dagger}(z))|0\rangle$ , where  $j^{\mu}$  is the electric current. Nevertheless, this derivation does not make use of the gauge symmetry whatsoever. Does this mean that the identity above does not require gauge invariance after all?

(Gergely Fejős)

35. In this exercise, we investigate an alternative Universe. This Universe contains three spatial and one time dimensions, and space can be described as a large cuboid. The lengths of the sides of the cuboid are denoted by  $L_x, L_y$  and  $L_z$ , its volume is V. The Creator prescribed cyclic boundary conditions.

The alternative Universe is filled with a Bose-Einstein condensate which consists of ultracold atoms. The condensate is characterized by a complex wavefunction  $\Psi(\mathbf{r},t)$ . The dynamics of the wavefunction is described by the so called Gross-Pitaevskii equation:

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) = \left(-\frac{\hbar^2}{2m}\Delta - \mu\right)\Psi(\boldsymbol{r},t) + g\cdot\Psi^*(\boldsymbol{r},t)\Psi(\boldsymbol{r},t)\Psi(\boldsymbol{r},t)\;,$$

where  $\hbar$  is the reduced Planck-constant, m is the mass of the ultracold atoms,  $\mu$  is the chemical potential and g is a constant which characterizes the atomic collisions. The number of condensed atoms is denoted by  $N_c$ . The local density of the condensate is given by the absolute value of the wavefunction squared. Let  $c = \sqrt{\frac{g}{m} \frac{N_c}{V}}$  denote a parameter which has the dimension of velocity and let  $\xi = \frac{\hbar}{mc}$  denote another parameter, which is small, and which has the dimension of length.

The alternative Universe is habited by intelligent creatures and the creatures live in two colonies, whose distance is large. The habitants of the two colonies communicate by creating small wavelike perturbations in the otherwise homogeneous condensate. Due to the limitations of the moderately advanced technology of the creatures, they can only create waves in the condensate, whose wavelength is much larger than the length  $\xi$ .

In one of the colonies, the leading physicists of the creatures are arguing about the fundamental nature of the physical laws of their world. While Alice says that the aether should exist, Bob argues that the aether hypothesis is not needed, because their world is fundamentally Lorentz invariant.

Who is right, Alice or Bob?

To decide who is right in the controversy, let's write up the equation of the small perturbations of the complex wavefunction, and then let's make a real wave equation from this complex equation, which will contain derivatives higher order than the previous complex one. In this equation, the parameters c and  $\xi$ , which were introduced above, will appear. Let's analyze this equation from the perspective of the debate.

(Gábor Kónya)

36. Dr. Absoluto Zero, the beloved eternal dictator of the small, very self conscious equatorial country, Rubbercoast, was summoning his Prime court physicist. Dr. Ali Whyheknows stepped close to the huge desk with a sigh.

'It is unbelievable! The president of Radiria has built a Light-green Presidential Residence twice larger than my own Pink Palace in Port Goomy!'

The Prime Physicist got a bit frightened. This is not the first time that his scientific knowledge needs to serve the decade long clashes with the other small but proud equatorial country. And he was right, as the dictator continued:

'I can not tolerate this. Tomorrow you need to launch our intercontinental ballistic missile, named after dr. Absoluto Zero, and get that Presidential Residence in Radiria destroyed!'

'But Mr. President, with your permission! Three years ago you have canceled all the rocket projects and you ordered the engineers and technicians to harvest the gummy palms, furthermore you passed the fuel of the rockets to the National Helicopter Fleet of the Ministry of Public Enlightenment and Propaganda.'

'I want solutions from you scientists, not just chatter! Some time ago you told me that according to a new scientific theory, the Earth rotates, right?'

'Yes, but...'

'Well then take that warhead, throw it high up vertically, and then the Earth turns under it. If you do right, it will fall right onto Radiria!'

'I was just saying we have no rocket...'

'Never mind. I saw that cool film starring Bruce Willis dealing with an asteroid... do so: make a nice big piece of rock or artificial comet and shoot it vertically to the sky! Make sure that it falls directly to Radirbourg.'

'Yes President! On the Gummy Harvest Celebration Day, we will launch it ceremoniously, and few hours later it will fall on Radiria...'

'No that's not quite fine. If a meteorite falls on Radiria on the day of the Gummy Harvest, Radio Free Gummy will suspect us... I want the following: the stone should be flying for a day, and as on that day after the ceremony I will be hunting gummy squirrels in the gummy palm forest, nobody can blame us being involved!'

The Prime Physicist, dr. Ali Whyheknows started out with his calculations. Fortunately, air drag needs not to be taken into account, since that was banned, along with all other forms of resistance, being physical, social or political, as we know since the 1987 Ortvay Competition.

On the next day, the Prime Physicist showed the results to the President, as a nice 3D animated presentation: the gloomy artificial meteorite leaves Port Goomy, changes its trajectory according to the laws of gravity, and then after flying for one day, it hits Radiria.

The President was not impressed.

'You should be aware: we talk straight, we shoot straight. If we throw a stone, we throw it straight up! When my dear father, Absoluto Minus One took me to the gummy woods to hunt for gummy squirrels, he showed me how to throw a boomerang: straight upwards! Make sure that the projectile starts vertically, and the rest is taken care of by the Earth: let it turn Radiria under it.'

The Prime Physicist made the necessary modifications, and indeed, the simulation showed the projectile leaving Earth straight up, and then nicely falling back to Radiria.

'That is right! Do not change the program a bit. Our diligent nation will be victorious!'

And so it happened: the diligent people of Rubbercoast started to work hard. After a few months they created an enormous artificial meteorite, and using a full year's harvest of gummy palms and wood from felling forests of gummy palm trees, they created the world's largest gummy slingshot. The device was completed on the Gummy Harvest Celebration Day.

During the ceremony, thousands of volunteers (ordered by the State) tensioned the slingshot, which projected the huge stone: vertical as proper. Meanwhile the National Gummy TV communicated that the upthrown stone symbolizes how the standard of living of the people of Rubbercoast is continuously increasing under the leadership of dr. Absoluto Zero.

The dictator traveled to the woods of the gummy squirrels with satisfaction.

Next day unpleasant news arrived: a large, artificially-looking meteorite fell down and crushed the Pink Palace in the city of Port Goomy to pieces.

Our question is this, then: What is the distance between the capital of Radiria and Port Goomy?

(Gyula Dávid)

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